

## **Advisor's report on the PhD. thesis of Tomáš Ebenlendr** ***Combinatorial algorithms for online problems:*** ***Semi-online scheduling on related machines***

This thesis studies online and semi-online algorithms for scheduling. In this wide area, it focuses on the minimization of makespan (the length of the schedule) on uniformly related machines.

The main part of the thesis, Section 2, is a very complete and successful study of preemptive scheduling, i.e., of the case when a job can be split between different machines. This is a classical problem in scheduling theory. Due to the preemptions, the problem becomes tractable: not only it is polynomial-time computable but also the structure of the optimum is very well understood. Yet before the work of this thesis, the understanding of the corresponding online problem was very partial. A typical paper considered a very special case of two machines and gave an analysis of the optimal competitive ratio depending on the speeds (or rather on the ratio of the speeds which is a single continuous parameter), using an extensive ad hoc analysis. In contrast, this thesis, fully using both previous insights concerning the optimum and new techniques, gives a universal algorithm – not only for any speeds but also for essentially all semi-online restrictions studied in the literature – and proves its optimality. Moreover, it provides a systematic way of deriving full analysis of the optimal competitive ratios for a small number of machines. This shows the power of the new techniques: the move from two to three or four machines requires handling two or three parameters instead of one, which creates a considerably more complex situation. I expect that these results will remain the state-of-the-art for a number of years.

The shorter Section 3 also contains a very interesting result. It studies the considerably harder problem of non-preemptive scheduling. It proves a new lower bound, improving the previous result from published in the year 2000.

The results of the theses leave several important problems open. One area is improving the overall ratios in the preemptive case – it is very intriguing that despite of our new understanding, we are not able to give an optimal bound on the competitive ratio over all combinations of speeds. Another area is non-preemptive scheduling where the new result is incremental and further improvements should follow.

The results of Section 2 were published in several joint papers (with me and one also with another student Wojciech Jawor) in major conferences and journals; the last one was invited to and published in the special volume of selected papers from STACS 2009. In addition, the results for specific restriction were presented in a single-authored paper at the conference PPAM. The results of Section 3 are not published yet. The candidate has also additional good publications not connected to the topic of the thesis.

The presentation of the thesis could be improved esp. in the introductory part. There are some problems with inconsistent notations and also some small grammatical errors. Clearly, this reflects the fact that these parts were written with some time gap from the original presentation. The quality of presentation of the technical parts is better, which is natural as they are based mostly on published and refereed papers. In particular, I appreciate the presentation of the cases for the small number of machines. The way of presenting them was invented and polished quite independently by the candidate.

Also in terms of scientific contribution the part with specific cases is the most independent work of the candidate. This part involves a significant number of computer-assisted proofs, both for the numerical solutions and for solving the cases. This involves quite ingenious techniques of programming with the symbolic manipulations to allow for automated generation of most of the cases. This limit the amount of the manual work to a small fraction – and without this handling such a number of cases would have been impossible. Yet it should be stressed that the resulting proofs can be checked without computer assistance. As a result, this part does not stop at a sample of results but gives a very good coverage of models studied in the literature.

The strongest aspect of the thesis is the quality of the results and the comprehensive, in-depth study of the topic in an active area of research. The number and quality of the papers presenting the results is very good. In my opinion, the candidate showed the ability of performing independent research and presenting its results. I strongly recommend awarding the PhD degree.

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